**Homework 11**



**P10.1.9** Determine **VO** in Figure P10.1.9.

**Solution:** Starting with **VO** and I, and working backwards, the currents and voltages are as shown. From



KVL around

the mesh

on the left,

 + 72**I** + 16**I** +

 = 0. From KVL around the loop consisting of the primary of the first transformer and the two 20 Ω resistors, 16**I** +  + **VO** – 20**I** + **VO** = 0. This gives . Substituting in the first KVL and simplifying, . It follows that **VO** = 2.58∠180° V.

**P10.1.11** Derive TEC looking into terminals ‘ab’ in Figure P10.1.11.



**Solution:** On open circuit, no current flows. The primary voltage is **VSRC** of the polarity shown. It follows that **VTh** = -4**VSRC** + **VSRC** = -3**VSRC** = -3∠0 V.

When the source is set to zero, the impedance looking into terminals ‘ab’ is 16×2 = 32 Ω.



**P10.1.16** Determine *ZX* in Figure P10.1.16 so that **VO** = 0.

**Solution:** Let **I** be the current through the secondary winding. This current flows through *ZX*, since the current in *ZL* is zero. The current in the primary winding is *a***I** in the direction shown. From KVL, *jωLa***I** + *ZX***I** = 0, which gives: *ZX* = -*jωLa.*



**P10.1.22** Derive TEC between terminals ‘ab’ in Figure P10.1.22.



**Solution:** Remove the 10 Ω resistor, to be inserted later. A primary voltage and secondary current may be assigned as shown. From KVL in the secondary circuit, 2**V1** = (40 + *j*20)**I**, or **V1** = (20 + *j*10)**I**. From KVL in the primary circuit, 20 = 2**I**(5 – *j*5) + **V1**. Substituting for **V1**, 20 = 10(1 – *j*)**I** + 10(2 + *j*)**I**; this gives **I** = 2/3 A. Hence, **VTh1** = V. This gives **VTh1** = -40/3 V.



Let terminals ‘ab’ be short-circuited.

The currents and voltages will be as shown. **V1** = 40(**I** + **Isc**) + *j*10**I**,

2**V1** = 40(**I** + **Isc**) + *j*20**I**; eliminating V1

Between these

equations, **I** = -**Isc**.

On the primary side, 20 = 5(2**I** + **Isc**) + 40(**I** + **Isc**) = 50**I** + 45**Isc** = -5**Isc**, so that **Isc** =

-4 A. It follows that *RTh*1 = 10/3 Ω.



When the 10 Ω resistor is reinserted, **VTH** = **Vab** =  V, and *RTh* = 10||(10/3) = 2.5 Ω.

**P10.1.31** Derive TEC looking into terminals ‘ab’ in Figure P10.1.31.



**Solution:** On open circuit, from the volts per turn, ;

For zero mmf: 300**I2** + 200**I1** = 0.

also, **V2** = 10 – 50**I2** and



Substituting for **V1**, . Substituting **I1**, , or 50**I2** = *j***V2**. Substituting 50**I2**, **V2** = 10 – *j***V2**, or  V, and **VTh** =



2**V2** = 10(1 – *j*) V.

When a test source **VT** is applied, with the independent source set to zero, **VT** = 2**V2**. **I2** = **V2**/50 = **VT**/100 . **V1** = **VT**/5 = -*j*8**I1**, or **I1** = *j***VT**/40. The mmf equation is: 500**IT** + 300(**IT** – **I2**) +200(**IT** – **I2** – **I1**) = 0, or 10**IT** – 5**I2** – 2**I1** = 0, Substituting **I1** and **I2** in terms of **VT** gives:



, or:

 Ω = *ZTh*.

**P10.1.35** Derive TEC looking into terminal ‘ab’ in Figure P10.1.35.



**Solution:** **V2** = *j*100**I2**, **VTh** = −*j*40**I3**, 50 = 50**I1** + 0.5**V2** – 2**VTh**. Moreover, **I1** = 2**I2** and **I3** = -2**I1** = -4**I2**. Substituting for **I1** and **V2** in terms of **VTh** gives: 50 =**VTh** + **VTh** – 2**VTh**; or, 50 = **VTh**, or, **VTh** =



-26.1 + *j*9.65 V.

Let a test voltage source be applied between terminals ‘ab’. The *j*100 Ω impedance reflected to the primary side is *j*25 Ω. **I1** ; **I3** =



-2**I1**, and **IT** = **IC** – **I3** = . It follows that  15.44 + *j*1.69 Ω.

**P10.2.2** Determine *k* in Figure P10.2.2 so that no current flows in *ZX*.



**Solution:** With no current flowing in *Zx*, **I** = **VI**, and **V1** =**VI** = **VI**. Moreover, **V2** = **VI**.



Since **V1** = **V2**, then, *M* = mH. It follows that .



**P10.2.5** Determine a in Figure P10.2.5 so that *Yin* = 0, assuming *ω* = 1 Mrad/s.

**Solution:**The two coupled coils have *Leq* = 6 + 4 – 2×3 = 4 μH, and an impedance of *jωLeq* = *j*4 Ω *;* 1/*jωC* = 1/(*j*106×0.25×10-6) = -*j*4 Ω.

When a test source **VT** is applied, the test current **IT** should be zero. The voltage across *jωLeq* is (**VT** – *a***VT**), so that **I** = (1 – *a*)**VT/**(*j*4). From KCL at the upper node, *a***I** = **I** + **VT/**(-*j*4), or **I** = **VT/[**(*j*4)(1 – a)]. Equating the two expressions of **I**:



, or , *a* = 1 ± 1, or *a* = 2.